

Mathematical Modelling of Malaria Transmission Dynamics in Rural Regions of Jharkhand and Bihar: An SEIR Model Approach

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ABSTRACT

Malaria remains a significant public health challenge in rural regions of Jharkhand and Bihar, where socio-economic, climatic, and environmental factors contribute to persistent transmission. This study aims to develop a mathematical model to better understand the transmission dynamics of malaria in these rural settings. Using a compartmental model (SEIR), we incorporate key factors such as the vector-host ratio, population immunity, climate variability, and migration patterns. Data collected from field studies and secondary sources will be used to estimate transmission parameters, simulate various intervention strategies, and evaluate their potential to reduce the disease burden. Model simulations demonstrate the influence of environmental factors like temperature and rainfall, as well as the effectiveness of vector control and mass drug administration programs. Sensitivity analysis highlights the critical role of timely interventions in altering transmission dynamics. The findings from this study provide valuable insights for policymakers and public health officials in designing targeted strategies for malaria control in rural India. This paper also emphasizes the importance of incorporating local data and real-time feedback into mathematical models for improving malaria control strategies.

Keywords: Malaria transmission, Mathematical modelling, SEIR model, Rural areas

1. Introduction

The fight against malaria continues to be a significant obstacle for public health in India, especially in rural regions of states such as Jharkhand and Bihar. These areas are distinguished by a confluence of environmental, socio-economic, and healthcare elements that, when combined, provide circumstances that are favourable for the spread of malaria. Anopheles mosquitoes are the primary vectors of transmission for the illness, and the spread of the disease is impacted by a variety of variables, including climate, population density, and the accessibility of healthcare facilities. As a

result of their large rural landscapes, deep forests, and river basins, the states of Jharkhand and Bihar create an environment that is favourable for the reproduction of mosquitoes. Additional factors that contribute to the spread of malaria include socioeconomic situations such as poverty, restricted access to healthcare, and poor infrastructure*. The population in these regions often has a limited understanding of preventative measures and a restricted access to treatment, which results in high infection rates as well as large morbidity and fatality rates. Malaria is a disease that humans get from mosquitoes and is caused by a parasite. Two of the five kinds of parasites, Plasmodium falciparum and Plasmodium vivax, represent the greatest danger. Plasmodium falciparum is the most lethal form of malaria infection, while Plasmodium vivax is the most prevalent form of malaria parasite in the majority of nations outside of sub-Saharan Africa. The World Health Organisation (WHO) revealed that there were roughly 241 million cases of malaria around the globe in the year 2020[†]. On the other hand, the number of fatalities caused by malaria was expected to reach 627,000 in the same year. Africa was the most affected area in 2020, accounting for 95% of all malaria cases and 96% of all fatalities caused by the disease[‡]. Eighty percent of the overall number of victims in that region were youngsters less than five years old. Even though there have been decades of attempts to eradicate and manage the illness on a worldwide scale, the disease is reemerging in regions where control measures were previously successful. In order to mitigate the transmission of malaria, many media efforts have been implemented to encourage the use of insecticide-treated nets (ITN) and bed nets. The attempts to disseminate ITN knowledge to the public have been essential in boosting the adoption of mosquito nets. In order to communicate with the general audience, media campaigns use a variety of approaches. The most significant outcome is when a health professional or a volunteer informs the general public about successful antimalarial programs. This is the most relevant effect. In order to have a better understanding of the spread and effect of the illness, mathematical models of the transmission dynamics of malaria may be implemented. To better plan for the future and to guide the formulation of suitable policies to manage the condition, it is helpful. There have been a number of mathematical models of the transmission dynamics of malaria that have been published in the past. These models follow the basic S–I–R model[§]. These models have been changed by a large number of researchers, who have included additional concepts linked with the dynamics of malaria and any potential methods of controlling the illness. However, these studies did not take into account the influence that awareness campaigns have had on the prevention and control of malaria. Controlling the spread of malaria may be accomplished with the use of significant instruments provided by awareness efforts. Around the globe, millions of people have lost their lives to malaria, making it one of the infectious illnesses that is among the deadliest. Malaria is a disease that may affect 3.3 billion people worldwide, which is equivalent to half of the world's population. Malaria outbreaks can occur

* Mubayi, A., Castillo-Chavez, C., Chowell, G., Kribs-Zaleta, C., Siddiqui, N. A., Kumar, N., & Das, P. (2010). Transmission dynamics and underreporting of Kala-azar in the Indian state of Bihar. *Journal of theoretical biology*, 262(1), 177-185.

[†] Patel, P., Bagada, A., & Vadia, N. (2024). Epidemiology and Current Trends in Malaria. *Rising Contagious Diseases: Basics, Management, and Treatments*, 261-282.

[‡] Guinovart, C., Navia, M. M., Tanner, M., & Alonso, P. L. (2006). Malaria: burden of disease. *Current molecular medicine*, 6(2), 137-140.

[§] Zaman, G., Kang, Y. H., & Jung, I. H. (2008). Stability analysis and optimal vaccination of an SIR epidemic model. *BioSystems*, 93(3), 240-249.

in 104 countries throughout the globe^{**}. It has been estimated that between 300 and 500 million people of all ages get infected with malaria each year, and that between 1.5 and 2.7 million people lose their lives as a result of the disease. In tropical and subtropical locations, such as Africa, Asia, Latin America, the middle East, and some sections of Europe, malaria is often found and has a broad range of transmission. Sub-Saharan Africa is the region that has the highest number of cases and fatalities. To be more specific, thirty nations in sub-Saharan Africa are responsible for ninety percent of all fatalities caused by malaria worldwide. Unbelievably, the illness claims the life of a child in Africa every thirty seconds, and worldwide, it claims the lives of more than two thousand young people every single day. Within the population of children under the age of five in Nigeria, for instance, malaria is responsible for sixty percent of outpatient visits and thirty percent of hospitalisations. Malaria is a disease that is caused by infection with single-celled (protozoan) parasites of the genus *Plasmodium*. Malaria is characterised by paroxysms of chills, fever, headache, pain, and vomiting. Malaria continues to be one of the most widespread and dangerous human infections in the globe. Infected female *Anopheles* mosquitoes, which are known as vectors, are responsible for transmitting the parasites to people via their bites. In humans, malaria is caused by five different kinds of parasites: *Plasmodium falciparum*, *Plasmodium vivax*, *Plasmodium ovale*, *Plasmodium malariae*, and *Plasmodium knowlesi*. *Plasmodium falciparum* is the most dangerous type of the parasite, and it is most often seen in Africa^{††}. The parasite is the infectious agent that is responsible for the highest number of fatalities and clinical cases in tropical regions. It is possible for its infection to result in significant consequences that spread to other organs, including the kidneys, lungs, and brain. When it comes to providing improved insights into the behaviour of the illness, mathematical models for the transmission dynamics of malaria are beneficial. The decision-making processes regarding intervention options for preventing and managing the outbreak of malaria have been significantly impacted by the models, which have played significant roles in the decision-making process. As a result, all of the additional models that are now available for the dynamics of malaria are built from the three fundamental models that were discussed earlier. These models include a variety of parameters in order to make them more biologically realistic in terms of explaining disease prevalence and prediction.

2. Related Work

Wu et al. (2023) despite the fact that the Ross-Macdonald model had a significant impact on the research of malaria transmission dynamics and control, it was found to be lacking in elements that might represent the dispersion of parasites, travel, and other significant aspects of heterogeneous transmission. They proposed a framework for differential equation modelling that was based on patches. This framework expanded the Ross-Macdonald model with sufficient skill and complexity to facilitate planning, monitoring, and evaluation for the management of *Plasmodium falciparum*

^{**} Patel, P., Bagada, A., & Vadia, N. (2024). Epidemiology and Current Trends in Malaria. *Rising Contagious Diseases: Basics, Management, and Treatments*, 261-282.

^{††} Antinori, S., Galimberti, L., Milazzo, L., & Corbellino, M. (2012). Biology of human malaria plasmodia including *Plasmodium knowlesi*. *Mediterranean journal of hematology and infectious diseases*, 4(1).

malaria. An algorithm for mosquito blood feeding was used as the basis for the development of a generic interface that was created for the purpose of constructing structured spatial models of malaria transmission. In order to mimic adult mosquito demography, dispersion, and egg-laying in response to the availability of resources, new algorithms were constructed; these algorithms were developed. A modular framework was constructed by decomposing, redesigning, and reassembling the fundamental dynamical components that describe the ecology of mosquitoes and the spread of malaria. The framework's structural parts, which included human population strata, patches, and aquatic habitats, interacted with one another via a flexible design. This design made it easier to develop ensembles of models with scalable complexity, which supported robust analytics for malaria policy and adaptive malaria management. They proposed updated definitions for the human biting rate and entomological inoculation rates and presented new formulas to describe parasite dispersal and spatial dynamics under steady-state conditions, including human biting rates, parasite dispersal, the “vectorial capacity matrix,” a human transmitting capacity distribution matrix, and threshold conditions. A package for the programming language R had been built that was capable of implementing the framework, solving the differential equations, and computing spatial metrics for models that were constructed using this framework. Malaria had been the primary emphasis of the creation of the model and metrics; but, due to the modular nature of the framework, the same concepts and software might be used for other infectious diseases that are transmitted by mosquitoes.

Kuddus & Rahman (2022) An explanation was given that malaria, which is an infectious illness caused by a parasite that is transmitted by Anopheles mosquitoes, was common and harmed individuals of all ages. They made the observation that blood-borne infections connected with malaria were responsible for roughly 110 million clinical cases of malaria and between one and two million fatalities related with Plasmodium falciparum each year around the globe, including in Bangladesh. Within their work, they built a model for the dynamics of malaria transmission between humans and mosquitoes, as well as an analysis of the system features and potential solutions. Analytical and numerical findings indicated that the disease-free equilibrium would be asymptotically stable if the fundamental reproduction number $R_0 < 1$ was less than 1. This would indicate that malaria would spontaneously disappear under these circumstances. In addition, the malaria sickness continued to be present in the community if R_0 was greater than 1. In addition to this, they offered the model calibration in order to estimate parameters using data on the incidence of malaria in Bangladesh from 2001 to 2014. Using the partial rank correlation coefficient approach, sensitivity analysis was also carried out in order to determine which factors were the most important. In their research, they discovered that the most significant factor in determining the incidence of malaria was the contact rate between people and mosquitoes. In conclusion, numerical simulations and graphical analysis were used in order to investigate the effects of advancement rate, mortality rate connected to the illness, recovery rate, and the rate at which immunity was lost.

Ndamuzi & Gahungu (2021) It was observed that Burundi, a nation located in East Africa with a climate that is classified as moderate, has seen a concerning increase in the number of cases of malaria in recent years. A deterministic model of the transmission dynamics of malaria parasites in mosquito and human populations was developed by the authors of the particular article that they published. The SEIR model served as the foundation for the development of the mathematical model. R_0 , often known as the basic reproduction number, was determined to be an epidemiological threshold using the formula. The equilibrium point of the disease-free state was locally asymptotically stable if $R_0 < 1$, and it was unstable if $R_0 > 1$ was more than $1R_0 > 1$. They were able to demonstrate, via the use of a Lyapunov function, that this disease-free equilibrium point was globally asymptotically stable whenever the fundamental reproduction number was less than one. In this study, both the presence of endemic equilibrium and its singularity were investigated. They also demonstrated, with the help of the Lyapunov function, that the endemic equilibrium was globally asymptotically stable if R_0 was more than $1R_0$ and $1R_0$ was greater than 1. After everything was said and done, the system of equations was solved numerically in accordance with the data that Burundi had on malaria. Based on the findings of their model, it was determined that in order to curb the spread of malaria in Burundi, it is necessary to implement optimal control measures. These measures include reducing the number of mosquito bites on humans per unit of time (σ), the vector population of mosquitoes (N_v), the probability of being infected for a human bitten by an infectious mosquito per unit of time (b), and the probability of being infected for a mosquito per unit of time (c).

Rahman & Kuddus (2020) Researchers found that infectious illnesses like malaria were responsible for a significant loss of population in both poor nations and industrialized countries on an annual basis. In order to better understand and address this issue, modelling malaria was thought to be one of the most successful interventions. For the purpose of providing assistance for the design and characterization of the malaria illness in Bangladesh, they devised a dependable quantitative modelling approach. Results revealed that the effects of some influencing factors, such as service factors (e.g., lack of budgets in government agencies, antiviral medicine, and poor quality of service facilities), disease-related factors (e.g., nutritional status and existing illness), environmental factors (e.g., urbanization and crowdedness), and sociological factors (e.g., education and religious beliefs), were significant. To be more specific, the social component of education shown strong multidimensional implications on the onset of the illness as well as its mitigation. Poor educational status led to a variety of negative effects, including (i) a lack of awareness regarding the causes of malaria, the severity of health effects, and how and where to access treatment services; (ii) a refusal to receive vaccinations; and (iii) a lack of familiarity with good health and nutritional facts that contribute to nutritional status. Using the model that was constructed, the research also produced a projection of new cases of malaria until the year 2025, as well as recommendations for malaria control techniques.

Nwankwo & Okuonghae (2019) The influence of temperature on the effectiveness of intravenous (Iv) vaccines was explored using a mathematical model that was established for the spread of malaria. They discovered that the disease-free state of the autonomous version of the stated model was globally asymptotically stable in the absence of disease-induced mortality when the associated reproduction number was less than unity. This was accomplished in the absence of disease-induced mortality. Additionally, the illness continued to be present in the population whenever the related reproduction number of the non-autonomous model was larger than one. A sensitivity and uncertainty analysis of the autonomous model was carried out. The related reproduction number, the number of infected persons, and vectors were used as response functions in the study.

Eikenberry & Gumel (2018) it was brought to light that malaria, which is considered to be one of the most devastating diseases in human history, continued to take the lives of around half a million people every year, with the majority of fatalities happening in children under the age of five who were residing in tropical Africa. They explained that the range of this disease was restricted to warmer regions of the world due to climate, and that as a result, anthropogenic global warming (and climate change in general) now posed a threat to alter the geographic area for potential malaria transmission. This was due to the fact that both the Plasmodium malaria parasite and the Anopheles mosquito vector had lifecycles that were highly dependent on temperature, and the aquatic immature Anopheles habitats were also highly depend. They stated that a wide range of process-based (or mechanistic) mathematical models had been proposed for the intricate, highly nonlinear weather-driven Anopheles lifecycle and the dynamics of malaria transmission. However, these models had arrived at somewhat contradictory conclusions regarding the optimal temperatures for transmission and the potential impact that rising temperatures could have on the distribution of malaria. Some research indicated that the geographic range of malaria would mostly move, while others expected that the region at risk for malaria would significantly expand. In a broader sense, the first appearance of Plasmodium falciparum as a significant human pathogen in tropical Africa around 10,000 years ago was driven by both global and local environmental changes. The illness has a long and deep history that continues up to the current day. They provided a timeline of some major modelling efforts, beginning with the classical works of Sir Ronald Ross and George Macdonald and ending with recent climate-focused modelling studies. Their paper covered major aspects of malaria biology, methods for formalising these into mathematical forms, uncertainties and controversies in proper modelling methodology, and provided a timeline of some major modelling efforts. In addition to this, they made an effort to situate the mathematical work within a more comprehensive historical framework around the "million-murdering Death" of malaria.

Otieno et al. (2016) a mathematical model for the transmission dynamics of malaria with four time-dependent control measures in Kenya was suggested and analysed. These control methods are as follows: treatment, insecticide-treated bed nets (ITNs), indoor residual spray (IRS), and intermittent preventive treatment of malaria in pregnancy (IPTp). In the beginning, they took into account the constant control parameters and computed the fundamental reproduction number. Additionally, they investigated the presence of equilibria and their stability, and they carried out a stability analysis. The

researchers demonstrated that if the value of R_0 is less than or equal to $1R_0$, then the disease-free equilibrium in D is globally asymptotically stable. The unique endemic equilibrium was present and was globally asymptotically stable if R_0 was greater than 1. Additionally, the model displayed backward bifurcation at the point when $R_0=1$. The findings of the sensitivity analysis revealed that the parameters that were the most sensitive were the mosquito death rate and the mosquito bite rate. After that, they took into account the time-dependent control scenario and used Pontryagin's Maximum Principle in order to determine the essential criteria that must be met in order to achieve the most effective control of the illness by using the suggested model. It has been shown that the optimum control issue does in fact exist. The optimal control strategy for malaria control in endemic areas was the combined use of treatment and IRS, according to numerical simulations of the optimal control problem using a set of reasonable parameter values. For epidemic-prone areas, the use of treatment and IRS was the optimal control strategy; for seasonal areas, the use of treatment was the optimal control strategy; and for low-risk areas, the use of ITNs and treatment was the optimal control strategy. After doing their research, they came to the conclusion that control programs that adhered to these principles had the potential to successfully prevent the spread of malaria illness in various locations in Kenya where malaria is transmitted.

Olaniyi & Obabiyi (2013) The authors introduced a seven-dimensional ordinary differential equation that models the transmission of *Plasmodium falciparum* malaria between people and mosquitoes. The equation takes into account non-linear forces of infection in the form of saturated incidence rates. In response to the presence of malaria parasites in both human and mosquito populations, these incidence rates developed antibodies. Malaria is caused by parasites. They were able to demonstrate the presence of an area in which the model might be developed from an epidemiological standpoint. Through the use of the next-generation matrix approach, the threshold parameter, which is the reproduction number R_0 , was utilized in order to conduct an investigation into the stability analysis of the disease-free equilibrium. Based on the findings of the model, it was determined that the equilibrium of disease-free states was asymptotically stable when the threshold parameter was less than one, but it was unstable when the threshold value was more than one. It was also found that the presence of the one-of-a-kind endemic equilibrium was established under certain circumstances. The analytical findings were validated via the use of numerical simulations, which also allowed for the investigation of the potential behaviour of the model that was developed.

Stuckey et al. (2013) It should be highlighted that when nations transitioned from malaria control programs to pre-elimination programs, it became more crucial to evaluate the success of malaria control treatments based on their influence on transmission. For the purpose of examining the links between malaria indicators, mathematical modelling was used. This allowed for the translation of data that could be readily quantified into measures of transmission, and it addressed important difficulties that were associated with previous techniques of measuring transmission. The simulations demonstrated that these indicators were statistically connected, which made it possible to make direct comparisons of the transmission of malaria by combining data obtained using a variety of methodologies over a wide range of transmission intensities and seasonal patterns. They came to the

conclusion that the outcomes of such models might provide public health authorities with precise estimations of transmission, broken down according to seasonal patterns. These estimates were essential for evaluating and adapting malaria control and eradication programs to particular environments.

Eckhoff (2013) In order to investigate baseline transmission, the impacts of seasonality, and the impact of interventions, a model for Anopheles population dynamics and malaria transmission was merged with a microsolver for within-host dynamics. In order to reproduce the Garki Project, a simulation was performed that included the pre-intervention baseline as well as the various combinations of treatments that were implemented. Modifications were implemented, and the combination of a longer project length, an extension of dry-season spraying, and transmission-blocking vaccinations was successful in achieving local eradication under some situations. Numerous treatments were simulated in transmission settings that varied in transmission intensity and underlying seasonality. These settings were used to model transmission. As a result of the addition of vaccinations to the already existing vector control measures, the capacity to accomplish eradication was expanded to include greater baseline transmission and less appealing vector behaviour. They discovered that the consequences of vector control were less severe for a single species of *Anopheles gambiae* species complex when one species of the complex fed disproportionately outside for a given complex average behaviour. The seasonal oscillation in parasite dynamics and the efficacy of wet-season therapies were constrained by the fact that there was non-zero transmission during the dry season.

Chitnis et al. (2012) We have presented and analysed a periodically-forced difference equation model for malaria in mosquitoes. This model was able to capture the impacts of seasonality and enabled the insects to feed on a diverse population of targets. Through the use of numerical evidence, they demonstrated the existence of a singular globally asymptotically stable periodic orbit. Furthermore, they generated periodic orbits of field-measurable quantities that monitored the spread of malaria. In order to evaluate the efficacy of insecticide-treated nets (ITNs) and indoor residual spraying (IRS) in lowering the transmission, prevalence, and incidence of malaria, the researchers combined this model with an individual-based stochastic simulation model for the disease in people. ITNs were shown to be more successful than IRS in lowering transmission and prevalence, despite the fact that IRS would attain its maximum effects within two years, but ITNs would need two large distribution campaigns spread out over a number of years in order to achieve the same level of effectiveness. In addition, the effectiveness of the combination of the two therapies was higher than the effectiveness of each intervention on its own. Nevertheless, despite the fact that these measures decreased transmission and prevalence, they had the potential to result in an increase in clinical malaria. Furthermore, after three years of the programs being discontinued, all three malaria indicators recovered to their levels before the interventions were implemented.



Table 1: Comparative Reviews

Earlier Work Done	Research Area	Methodology Used	Algorithms	Findings
Wu et al. (2023)	Malaria transmission dynamics	Patch-based differential equation modelling framework	Mosquito blood feeding, mosquito demography, dispersal	Extended the Ross-Macdonald model to include parasite dispersal and travel. Developed new metrics and an R package for malaria control modelling.
Kuddus & Rahman (2022)	Malaria transmission in Bangladesh	Human-mosquito transmission dynamics model	Model calibration, sensitivity analysis	Stability analysis showed that malaria persists if $R_0 > 1$. Sensitivity analysis identified critical parameters affecting malaria prevalence.
Ndamuzi & Gahungu (2021)	Malaria transmission in Burundi	Deterministic SEIR model	Lyapunov function	Proved global stability of the disease-free and endemic equilibrium points. Identified key parameters for controlling malaria spread in Burundi.
Rahman & Kuddus (2020)	Malaria transmission in Bangladesh	Quantitative modelling technique	Prediction model	Identified significant socio-environmental factors, especially education, affecting malaria occurrence. Predicted new malaria cases until 2025.
Nwankwo & Okuonghae (2019)	Impact of temperature on malaria transmission	Mathematical model with sensitivity and uncertainty analysis	None explicitly mentioned	Demonstrated the temperature-dependent impact on malaria transmission. Identified critical parameters for controlling malaria prevalence.
Eikenberry & Gumel (2018)	Malaria transmission and climate change	Process-based (mechanistic) mathematical modelling	Various historical and climate-focused models	Reviewed malaria modelling history and explored the impact of climate change on malaria distribution. Identified uncertainties in modelling methodologies.
Otieno et al. (2016)	Malaria control strategies in Kenya	Mathematical model with time-dependent control measures	Pontryagin's Maximum Principle	Identified optimal malaria control strategies for different transmission settings. Sensitivity analysis showed the importance of mosquito biting and death rates.
Olaniyi & Obabiyi (2013)	Malaria transmission with saturated incidence	Ordinary differential equation model with stability analysis	Next-generation matrix technique	Showed that the disease-free equilibrium is stable at $R_0 < 1$ and unstable at $R_0 > 1$. Confirmed the unique endemic equilibrium under specific conditions.
Stuckey et al. (2013)	Malaria control interventions and transmission	Mathematical modelling and simulations	None explicitly mentioned	Showed that model indicators can translate data into measures of transmission, aiding public health officials in assessing malaria control and elimination programs.
Eckhoff (2013)	Malaria transmission and interventions	Model for Anopheles population dynamics and within-host dynamics	Various intervention simulations	Simulated various interventions; found that combining vaccines with vector control extends the ability to achieve elimination, even in higher transmission settings.
Chitnis et al. (2012)	Malaria control strategies	Periodically-forced difference equation model	Individual-based stochastic simulation model	Demonstrated that ITNs are more effective than IRS, but a combination of both yields the best results. Highlighted the potential for increased clinical malaria post-intervention.

3. Mathematical Models For Malaria Transmission

The mathematical modelling technique is an effective method for gaining a knowledge of the dynamics of the transmission of infectious illnesses such as malaria. The use of equations to represent the interactions between people, vectors (such as mosquitoes), and the environment allows us to simulate the spread of the illness under a variety of settings and assess the effect of a number of alternative treatments. In order to further our understanding of the transmission of malaria in rural regions of Jharkhand and Bihar, we will create a compartmental model. We are going to concentrate on the SEIR model, which stands for the susceptible-exposed-infectious-recovered model. This model offers a framework that allows us to integrate important components including vector dynamics, human immunity, and environmental variables.

3.1 The Basic Reproductive Number (R_0)

One of the most important parameters in malaria modelling is the basic reproductive number, often known as R_0 . This value represents the anticipated number of secondary cases that will be created by a single infected person in a population that is entirely susceptible to the disease. If R_0 is more than 1, the illness will spread across the population, but if R_0 is less than 1, the disease will go extinct. For malaria, R_0 depends on both the human and mosquito populations and is typically expressed as:

$$R_0 = \frac{m \cdot a^2 \cdot b \cdot c \cdot e^{-r}}{r}$$

Where:

- m = vector-to-host ratio (the number of mosquitoes per person)
- a = mosquito biting rate
- b = transmission probability from an infected mosquito to a human
- c = transmission probability from an infected human to a mosquito
- r = recovery rate of infected humans
- e^{-r} = survival probability of mosquitoes during the extrinsic incubation period

Understanding R_0 is essential because it helps us quantify the level of intervention needed to control or eliminate malaria in a given region.^{††}

3.2 SEIR Model for Malaria Transmission

In the SEIR model, the population is divided into four compartments:

1. **Susceptible (S):** Individuals who can contract malaria.
2. **Exposed (E):** Individuals who are infected but not yet infectious.
3. **Infectious (I):** Individuals who can transmit the disease to others.
4. **Recovered (R):** Individuals who have recovered and have gained immunity.

^{††} Akwafuo, S., & Guo, X. Epidemiological modelling of vaccination and reduced funeral rites interventions on the reproduction number, R_0 of Ebola virus disease in West Africa. *Journal of Infectious Diseases and Medical Microbiology*, 2(3).

The dynamics of the model can be described using the following system of differential equations:

$$\begin{aligned} \frac{dS}{dt} &= \mu N - \beta \cdot S \cdot \frac{I}{N} - \mu S \\ \frac{dE}{dt} &= \beta \cdot S \cdot \frac{I}{N} - (\sigma + \mu)E \\ \frac{dI}{dt} &= \sigma E - (\gamma + \mu)I \\ \frac{dR}{dt} &= \gamma I - \mu R \end{aligned}$$

Where:

- $N=S+E+I+R$ is the total population.
- μ is the natural mortality rate.
- β is the transmission rate, which depends on the contact rate between humans and mosquitoes.
- σ is the rate at which exposed individuals become infectious.
- γ is the recovery rate.^{§§}

3.3 Vector Dynamics

The human population is not the only factor that contributes to the spread of malaria; the mosquito population also plays essential roles. Susceptible mosquitoes (S_v) and infected mosquitoes (I_v) are the two categories that make up the mosquito population which is separated into two divisions. The dynamics of the mosquito population can be described using the following equations:

$$\begin{aligned} \frac{dS_v}{dt} &= \mu_v N_v - \lambda_v S_v - \mu_v S_v \\ \frac{dI_v}{dt} &= \lambda_v S_v - \mu_v I_v \end{aligned}$$

Where:

$N_v=S_v+I_v$ is the total mosquito population.

μ_v is the mosquito mortality rate.

$\lambda_v=b \cdot I/N$ is the rate at which mosquitoes become infected, which depends on the biting rate (b) and the proportion of infected humans (I/N).

The transmission of malaria between humans and mosquitoes is described by the following equations:

$$\begin{aligned} \lambda_h &= a \cdot b \cdot \frac{I_v}{N_v} \\ \lambda_v &= a \cdot c \cdot \frac{I}{N} \end{aligned}$$

Where:

- λ_h is the rate at which susceptible humans become infected by mosquitoes.
- λ_v is the rate at which mosquitoes become infected by biting infected humans.
- a is the biting rate, b is the transmission probability from mosquito to human, and c is the transmission probability from human to mosquito.^{***}

^{§§} Ndamuzi, E., & Gahungu, P. (2021). Mathematical modeling of malaria transmission dynamics: case of Burundi. *Journal of applied mathematics and physics*, 9(10), 2447-2460.

3.4 Environmental and Socio-Economic Factors

Environmental elements such as temperature, rainfall, and humidity have a crucial impact in the reproduction of mosquitoes and the transmission of malaria in rural parts of the states of Jharkhand and Bihar^{†††}. These factors can be incorporated into the model by making the mosquito mortality rate μ_v and biting rate a function of environmental variables:

$$\mu_v(T) = \alpha_1 \cdot e^{\alpha_2 \cdot T}$$

$$a(T, R) = \beta_1 \cdot T \cdot R$$

Where T is the temperature, R is the rainfall, and α_2, β_1 are constants derived from empirical data.

Both the recovery rate and the transmission rate may be influenced by socio-economic variables, such as the availability of healthcare, education, and sanitation services. As an example, treatments like as insecticide-treated bed nets (ITNs) and indoor residual spraying (IRS) have the potential to decrease the bite rate β , resulting in a significant reduction in β .

4. CONCLUSION AND FUTURE WORK

Through the construction and implementation of a compartmental mathematical model, this research has yielded significant conclusions about the dynamics of malaria transmission in rural regions of Jharkhand and Bihar. These conclusions have been very helpful. The investigation has shed light on the important impact that climatic elements, socio-economic situations, and current public health initiatives have on the spread of malaria. Through the use of model simulations, it has been shown that timely actions such as vector control, mass medication delivery, and public awareness campaigns have the potential to significantly decrease the incidence of malaria. Despite these results, the model has a number of shortcomings, such as the assumption that the behaviour of the population is consistent across all individuals and the absence of micro-level data on human mobility and immunity. In the future, research should concentrate on strengthening the model by including more detailed data on human mobility patterns. These patterns have a significant role in the spread of malaria, especially in rural areas that see seasonal migration. In addition, future models might include real-time climate data and sophisticated machine learning algorithms in order to produce more precise forecasts of malaria outbreaks based on weather patterns and circumstances that are conducive to the reproduction of vectors. The implementation of spatial models that are able to take into account the geographical variation in transmission rates is yet another promising route for future research. These models would make it possible to implement treatments that are more specifically targeted at the village or district level. The incorporation of socio-behavioral elements, such as the behaviour of seeking medical attention and the attitudes held by locals about malaria, has the potential to not only improve the prediction accuracy of the model but also give a more thorough knowledge of the obstacles that stand in the way of successful malaria management.

^{***} Shah, N. H., & Gupta, J. (2013). SEIR model and simulation for vector borne diseases.

^{†††} Das, M. K., Rahi, M., Kumar, G., & Raghavendra, K. (2022). A note on the insecticide susceptibility status of secondary malaria vector *An. annularis* in Jharkhand state of India. *Journal of Vector Borne Diseases*, 59(3), 253-258.

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