



A MATHEMATICAL MODELLING OF FUZZY ENVIRONMENT USING A HEURISTIC APPROACH

Jitendra Kumar Pandey*, Dr. Sudhir Gupta

* Research Scholar, Dept. of Mathematics, Himalayan Garhwal University, Uttarakhand
Associate Professor, Dept. of Mathematics, Himalayan Garhwal University, Uttarakhand

ABSTRACT

The project scheduling is of particular importance among project management issues. With the passage of time and the progress of science and the emergence of concepts such as stochastic and non-stochastic uncertainties and the need to consider these concepts in scheduling and project management, Critical Path Method has also developed as a new method with fuzzy approach, preserving its basic concepts. In this paper, the literature on calculations of project scheduling network with fuzzy approaches is reviewed, subsequently; an algorithm for project scheduling with fuzzy time and resources is developed. This algorithm first calculates the latest start times of activities under fuzzy environment, then, construct a feasible schedule by using the parallel scheduling method. In order to show the effectiveness of the proposed algorithm, a project scheduling problem is solved with certain number of resources. Finally, an example was solved by the algorithm, considering fuzzy activity durations and resources.

Keywords:

*Fuzzy Scheduling;
Path Method;
Scheduling Scheme* *Project
Critical
Parallel*



INTRODUCTION

The project scheduling is one of the most important and widely used planning fields. The application of theories in practice and the extent of studies in this field, indicate its importance more than before. The project scheduling includes a wide range of issues and classifications. For each type of project scheduling problems, various methods have been proposed by several researchers. Since the project scheduling problems are known as NP-hard, different heuristic and metaheuristic algorithms are presented for solving these problems. Researchers tries to incorporate the real-world conditions such as uncertainty into the project scheduling models. However, in many practical problems due to lack of information or lack of access to data, accurate estimate of the parameters of the model is impossible. With the introduction of fuzzy theory and its applications in the planning and the need to consider these concepts in project scheduling, the Critical Path Method (CPM) were also developed retaining its original concepts. One of the new methods in this field is the Fuzzy CPM. The principles of this method are like the conventional CPM, except that instead of crisp numbers, fuzzy numbers are used for activity durations. This means that the project is scheduled based on forward and backward passes by using fuzzy numbers and the earliest start times and latest finish times of activities are calculated. The important parameters of the project scheduling are such as activity duration, levels of access to different resources, activities' requirements for resources etc. can be mentioned. In many researches, only activity duration is assumed to be uncertain. In this study, activity durations as well as resources are considered to be uncertain. This paper is structured as follows. A brief review of the related literature is conducted in section 2. In section 3, the proposed algorithm is presented under fuzzy activity durations and resources. Two numerical examples are given to show the validity of the algorithm in section 4. Finally, conclusions and recommendations for further research are made in section 5.

RESEARCH LITERATURE

After the introduction of fuzzy logic by Professor LotfiZadeh to the world of science, its applications quickly spread in various branches of science, including project management. Chanas and Kamburowski (1981) applied fuzzy logic to project scheduling problems [1]. They used time



intervals for project activity durations considering triangular fuzzy numbers. Lorterapong and Moslehi (1996) applied fuzzy durations to the CPM [2]. Yao and Lin (2000) used the CPM with the new ranking formula based on alpha-cut and splitting the time interval into two parts corresponding to α and $(\alpha-1)$ [3]. Chanas and Zielinski (2001) applied LR fuzzy numbers to the CPM [4]. Blue et al. (2002) used fuzzy logic theory and possibility theory in the CPM [5]. Yao and Lin (2003) proposed a new fuzzy ranking method based on fuzzy numbers interval [6]. Han et al. (2006) presented an algorithm for the simplification of the Fuzzy CPM with its application in an airport construction project [7]. Soltani and Haji (2007) presented a linear programming approach for the CPM considering trapezoidal fuzzy numbers [8]. Chen and Hsueh (2008) proposed a similar method to Haji and Soltani's method [9]. Kumar and Kaur (2010) combined the method introduced by Kaufman and Gupta with the technique proposed by Dobios and presented a new formula for fuzzy rankings [10]. 11.Shakeela and Gansean (2011) as well as Haji Yakhchali (2011) introduced new formula for ranking fuzzy numbers [11,12]. Morovatdar et al. (2013) proposed an algorithm for the Fuzzy CPM using piecewise numbers [13]. Elizabeth and Sujatha (2013) developed a new formula for ranking fuzzy numbers based on a new vision in simultaneous control of both random uncertainties (by risk management) and non-random uncertainties (by fuzzy ranking). However, their studies were merely theoretical, as they did not attempt apply their formula to an actual case study [14]. Chandra and Kumar (2014) developed fuzzy ranking in the FCPM [15]. Few studies were conducted on resource-constrained project scheduling problems (RCPSp) [17]. Hapke and Slowinski (1996) developed priority rules in sequential and parallel-based scheduling methods for fuzzy parameters [17]. Hapke and Slowinski (2000) applied Simulated Annealing algorithm to solve the fuzzy multi-objective RCPSp problems [18]. Wang and Kerre (2001) proposed a fuzzy approach to solve the problems with the objective of minimizing risk [19,20]. Wang (2004) used Genetic Algorithm to solve the RCPSp problem [21]. Bhaskar et al. (2011) proposed a new heuristic approach to solve the RCPSp with fuzzy durations based on priority rules. They used parallel scheduling scheme and proposed a new priority rule based on the critical path and resource demand [22]. At any decision point in parallel scheduling scheme, they determined feasible subset of activities according to precedence and resource constraints, then, they determined the priorities of the subsets. The subset with the highest priority is selected for scheduling at that decision point. Hassanpour et al. (2014) used Simulated



Annealing algorithm for project scheduling problem with fuzzy data [23].

THE PROPOSED ALGORITHM

First, the fuzzy latest start times of activities (LS) are calculated, since the parallel scheduling method, which is used in this algorithm, is based on the LSs. For this purpose, the improved backward pass method proposed by the Soltani and Haji [8] has been used. In this algorithm, Wang ranking method is applied to compare the fuzzy LSs [24]. The steps of the algorithm are as follows:

1. Perform forward pass calculation to find out the earliest start times of activities.
2. Use improved backward pass method to calculate the latest start times of activities.
3. In case of existence of fuzzy resources, apply the below method to deal with resource constraints:
4. Apply parallel scheduling method with the smallest LS as priority rule for activity selection.

After selecting activities based on the smallest LS priority rule, the precedence constraints and resource limitations. We compare the available resources with the amount of required resources for each candidate activity to be selected. All candidate activities are placed in a set, then the first activity is selected by using the priority rule. To select the next activity based on the resource constraints, we use improved inverse subtraction method instead of the conventional fuzzy subtraction method. For example:

$$A = (2,3,6) \quad B = (5,9,11) \quad B - A = (5,9,11) - (2,3,6) = (-1,6,9)$$

But in improved method, we have:

$$A + R = B \rightarrow (2,3,6) + (x,y,z) = \begin{cases} 2 + X = 5 \\ 3 + Y = 9 \\ 6 + Z = 11 \end{cases} \rightarrow (3,6,5) \quad (5,9,11)$$

As it is seen from the above calculations, the obtained number does not have the features of triangular fuzzy number; so, by adding the triangular conditions we have:

$$\begin{cases} 6 + Z = 11 \\ Y = \min(z, 3 + y = 9) \rightarrow (3,5,5) \\ x = \min(y, 2 + x = 5) \end{cases}$$



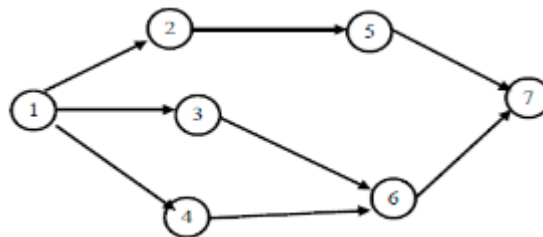
However, the number of available resources is considered as (3,5,5) and calculations for finding the second activity continues. It should be noted that the aforementioned method is only applicable for fuzzy resource amounts. Otherwise, the above calculations would not be required.

Numerical Examples

In this section, we provide two numerical examples in order to better understand the algorithm. First, to demonstrate the effectiveness of the algorithm in case of certain number of resources, the example given by Bhaskar [22] is solved by this algorithm. Then, a numerical example is provided with fuzzy activity durations and resources.

An example with certain resources

The example proposed by Bhaskar [22] is solved by this algorithm.



By converting the above problem from AON (Activity on Node) into AOA (Activity on Arc) network, the problem is defined as follows:

Table 1. The Data of The Examples Presented by Bhaskar [22]

activity	duration	resource	LS
(2-5)	(42:40:36)	17	(68:86:125)
(3-6)	(79:50:35)	12	(42:50:61)
(4-6)	(59:50:39)	3	(38:50:81)
(5-7)	(30:25:16)	13	(104:126:167)
(6-7)	(57:51:43)	17	(77:100:140)
(7-8)	(69:58:52)	16	(120:151:197)

By calculating LSs using the improved backward pass method, a feasible project schedule is obtained. Subsequently, Wang ranking method is used to compare fuzzy numbers [24]:

After having finished activity 1 (42,50,61), the candidate activities include:

$$R_{(1)} = \{(2-5), (3-6), (4-6)\}$$

$$LS_{(2-5)} = (68, 86, 125) \quad LS_{(3-6)} = (42, 50, 61) \quad LS_{(4-6)} = (38, 50, 81)$$

Comparing (2-5) and (3-6): Since 61 (b_u) is smaller than 68 (a_i), then, $LS_{(2-5)} > LS_{(3-6)}$

Comparing (3-6) and (4-6):

$$P(LS_{(3-6)} > LS_{(4-6)}) = \frac{(61 - 38)^2}{(61 - 38 + 50 - 42)(61 - 42 + 81 - 38)} = 0.275$$

As a result, the following relation exists:

$$LS_{(3-6)} < LS_{(4-6)}$$

Comparing (2-5) and (4-6):

$$P(LS_{(2-5)} > LS_{(4-6)}) = \frac{(81 - 68)^2}{(81 - 68 + 86 - 50)(125 - 68 + 81 - 38)} = 0.966$$

The following equation is obtained:

$$LS_{(3-6)} < LS_{(4-6)} < LS_{(2-5)}$$

The following equation is obtained:

$$LS_{(3-6)} < LS_{(4-6)} < LS_{(2-5)}$$

Due to resource constraints, first, activity (3-6) and then, activity (4-6) is scheduled. However, not enough resources have been left to carry out activity (2-5).

To reach the next decision point, activity (3-6) and (4-6) should be compared:

$$P(D_{(3-6)} > D_{(4-6)}) = \frac{(79 - 39)^2}{(79 - 39 + 50 - 50)(79 - 35 + 59 + 39)} = 0.966$$

The decision point two equals the finish time of activity (4-6) which is (81,100, 120), and the number of available resources at this time is 18. Therefore, only activity (2-5) starts at this time. To reach the decision point three, the duration of activities (2-5), (162 140 117) and (3-6) (140,100,77) are compared:



$$P(F_{(2-5)} > F_{(3-6)}) = \frac{(140 - 117)^2}{(140 - 117 + 140 - 100)(162 - 117 + 140 - 77)} = 0.922$$

Then, decision point three at time (77,100,140) (end of activity (3-6)) is available with 13 resources. Based on the precedence constraints, only activity (6-7) can be selected, but it remains unscheduled until activity (2-5) is accomplished due to resource constraints. The decision point four is at time (162,140,117). $R(t_4) = \{(5-7), (6-7)\}$

Since adequate number of resources are available for these two activities, both start at the intended time.

$$P(D_{(6-7)} > D_{(5-7)}) = 1$$

To start activity (7-8), we must wait until the end of activity (6-7). Therefore, activity (7-8) is started at (219,191,160) and finished at (288,249,212). The order and the times of the activities are exactly consistent with the result of Bhaskar algorithm [22].

Table 2. The Results of The Proposed Scheduling Algorithm

activity	Starting time	Finishing time
1	(0-0-0)	(61-50-42)
(4-6)	(61-50-42)	(120-100-81)
(3-6)	(61-50-42)	(140-100-77)
(2-5)	(120-100-81)	(162-140-117)
(5-7)	(162-140-117)	(192-165-133)
(6-7)	(162-140-117)	(219-191-160)
(7-8)	(191-160-219)	(288-249-212)

An Example of Fuzzy Time and Resources

In this section, the example proposed by Soltani and Haji [8] is solved under fuzzy durations and resources by using the proposed algorithm. The project network is shown in Fig. 2.

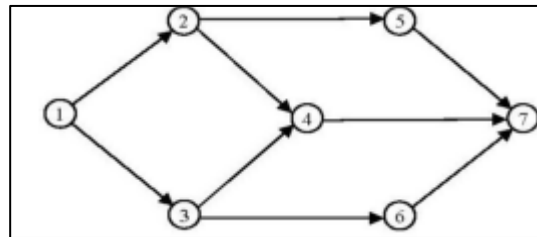


Fig. 2. Project Network of the Soltani and Haji [9]

Table 3: The Information of The Example Provided by Soltani and Haji [8] Taking into Account the Fuzzy Resources

activity	duration	Required resource
(1-2)	(25·28·32·35)	(8·11·13)
(1-3)	(40·55·65·70)	(6·8·12)
(2-4)	(32·37·43·48)	(7·8·9)
(3-4)	(20·25·35·40)	(6·7·8)
(2-5)	(35·38·42·45)	(6·9·13)
(3-6)	(42·45·55·60)	(10·13·14)
(4-7)	(60·65·75·85)	(10·16·20)
(5-7)	(65·75·85·90)	(4·6·8)
(6-7)	(15·18·22·26)	(6·8·10)

The number of available resources is (14,18,22). Activity durations are displayed as positive trapezoidal fuzzy numbers in Table 3. The start time is set as (0,0,0,0). By using the aforementioned equations, the main project indices can be calculated. These values are obtained as positive trapezoidal fuzzy numbers. Table 4 shows the earliest and latest start times for each activity.

Table 4. values of $E\tilde{S}_{ij}$ · $L\tilde{S}_{ij}$ calculated for the example provided by Haji and Soltani [8]

activity	ES	LS
(1-2)	(0·0·0·0)	(0·4·16·25)
(1-3)	(0·0·0·0)	(0·0·0·0)
(2-4)	(25·28·32·35)	(33·43·57·62)
(3-4)	(40·55·65·70)	(45·55·65·70)
(2-5)	(25·28·32·35)	(25·32·48·60)
(3-6)	(40·55·65·70)	(68·82·98·109)
(4-7)	(60·66·74·80)	(65·80·100·110)
(5-7)	(60·66·74·80)	(60·70·90·105)
(6-7)	(182·100·120·130)	(110·127·153·169)



At the beginning of the project, (0,0,0,0), we have: $R(0,0,0,0)=\{(1-2),(1-3)\}$ LS1-2=(0,4,16,25) LS1-3=(0,0,0,0) LS1-3

$$\begin{cases} 6 + X = 14 \\ 8 + Y = 18 \\ 12 + Z = 22 \end{cases} \rightarrow (8,10,10)$$

Now, we compare the available resources with the resources needed for activity (1-2): $A=(8,10,10)$ $B=(8,11,13)$

$$P(A > B) = \frac{(10 - 8)^2}{(10 - 8 + 11 - 10)(10 - 8 + 13 - 8)} = 0.190$$

As a result, the number of available resources is less than the amount of the required resources. Therefore, it is necessary to wait until the finish time of activity (1-3). The decision point two is achieved at (70,65,55,40) by finishing activity (1-3). At this time, the list of candidate activities is as follows:

$$R_2 = \{(3-4),(3-6),(1-2)\}$$

Now, the LS of the activities should be compared. LS 12=(0,4,16,25) LS 34=(45,55,65,70) LS 36=(68,82,98,109) Since a_u in LS1-2 is less than a_1 in both LS3-4 and LS 3-6, with regard to the second mode in comparisons, LS1-2 has the lowest amount. Then, activity (1-2) is scheduled at this time. At this stage, the amounts of LS3-4 and LS3-6 should be compared to select the second activity.

$$P(LS_{3-4} > LS_{3-6}) = \frac{(70 - 68)^2}{(70 - 68 + 82 - 65)(70 - 45 + 65 - 55)} = 0.002$$

Thus $LS_{3,4} < LS_{3,6}$

The amount of available resources at the time of (40,55,65,70) is as follows:

$$\begin{cases} 8 + X = 14 \\ 11 + Y = 18 \\ 13 + Z = 22 \end{cases} \rightarrow (6,7,9)$$



The amount of available resource is (6,7,9) and the amount of resource needed for accomplishing activity (3-4) is (6,7,8), Therefore,

$$\begin{aligned}
 &A=(6,7,9) \\
 &B=(6,7,8) \\
 &P(A > B) = \frac{(9 - 6)^2}{(9 - 6 + 7 - 7)(9 - 6 + 8 - 6)} = 0.60
 \end{aligned}$$

Activity (3-4) is scheduled at this moment.

$$S_{3-4} = (40,55,65,70) \quad F_{3-4} = (60,80,100,110)$$

Decision point three occurs when one of the two activities (1-2) and (3-4) is finished. Hence, we need to compare their

finish times.

$$\begin{aligned}
 &A=(65,83,97,105) \\
 &B=(60,80,100,110) \\
 &P(A > B) = \frac{105 - 60 + 97 - 80}{105 - 65 + 97 - 83 + 100 - 80 + 110 - 60} = 0.5
 \end{aligned}$$

Then, the two activities are finished at the same time. Decision point three is obtained at time (110,100,80,60), when the possible activities to be carried out are as follows:

$$\begin{aligned}
 &R_3 = \{(2-5), (3-6), (2-4)\} \quad AR = (14, 18, 22) \\
 &A = LS_{2,5} = (25, 32, 48, 60) \\
 &B = LS_{3,6} = (68, 82, 98, 109) \\
 &C = LS_{2,4} = (33, 43, 57, 62) \\
 &\text{as a result:} \\
 &P(A > B) = 0 \\
 &P(A > C) = \frac{60 - 33 + 48 - 43}{60 - 25 + 48 - 32 + 57 - 43 + 62 - 33} = 0.34 \\
 &P(B > C) = 1
 \end{aligned}$$

We have: $L S_{2,5} < L S_{2,4} < L S_{3,6}$
Hence, at time (110,100,80,60) activity (2-5) is scheduled.
 $S_{2,5} = (60, 80, 100, 110) \quad F_{2,5} = (95, 118, 142, 155)$

The amount of remaining resources is:

$$\begin{cases}
 6 + X = 14 \\
 9 + Y = 18 \rightarrow (8, 9, 10) \\
 12 + Z = 22
 \end{cases}$$



The number of resources needed to carry out activity (2-4) are (7,8,9)

$$A=(8,9,10) \quad B=(7,8,9)$$

$$P(A > B) = \frac{(9 - 8)^2}{(9 - 8 + 9 - 8)(10 - 8 + 9 - 7)} = 0.875$$

Then, activity (2-4) can be performed at time (60,80,100,110).

$$S_{2,4}=(60,80,100,110) \quad F_{2,4}=(92,117,143,158)$$

The amount of remaining resources:

$$\begin{cases} 7 + X = 8 \\ 8 + Y = 9 \\ 9 + Z = 10 \end{cases} \rightarrow (1,1,1)$$

Then, activity (3-6) cannot be performed at this time.

Decision point four is obtained by the end of each of activities (2-4) or (2-5).

$$A=F_{2,5}=(95,118,142,155)$$

$$B=F_{2,4}=(92,117,143,158)$$

$$P(A > B) = \frac{155 - 92 + 142 - 117}{155 - 95 + 142 - 118 + 143 - 117 + 158 - 92} = 0.5$$

The two activities are finished at the same time, then, the decision point four is achieved at time (155,142,118,95) with the following conditions.

$$R_4=\{(3-6),(4-7),(5-7)\} \quad AR=(14,18,22) \quad A=LS_{3,6}=(68,82,98,109) \quad B=LS_{4,7}=(65,80,100,110) \quad C=LS_{5,7}=(60,70,90,105)$$

$$P(A > B) = \frac{109 - 65 + 98 - 80}{109 - 68 + 98 - 82 + 100 - 80 + 110 - 65} = 0.508$$

$$P(B > C) = \frac{110 - 60 + 100 - 70}{110 - 65 + 100 - 80 + 90 - 70 + 105 - 60} = 0.615$$

As a result: $LS_{5,7} < LS_{4,7} < LS_{3,6}$

$$S_{5,7}=(95,118,142,155) \quad F_{5,7}=(160,193,227,245)$$

The amount of remaining resources is:



$$\begin{cases} 4 + X = 8 \\ 6 + Y = 9 \rightarrow (10,12,14) \\ 8 + Z = 10 \end{cases}$$

The amount of resource needed for activity (4-7) is as follows:

$$(20,16,10) \quad A=(10,16,20) \quad B=(10,12,14)$$

$$P(A > B) = \frac{(14 - 10)^2}{(14 - 10 + 16 - 12)(20 - 10 + 14 - 10)} = 0.857$$

So, activity (7-4) cannot be performed and it is necessary to be transferred to point five, at the end of activity (5-7).

Decision point five is achieved at time (245227193160) with the following conditions:

$$R_5 = \{(3-6), (4,7)\} \quad AR = (14,18,22)$$

Since $L_{S_{4,7}} < L_{S_{3,6}}$, activity (7-4) is scheduled at this time.

$$S_{4,7} = (160,193,227,245) \quad F_{4,7} = (220,258,302,330)$$

The amount of available resource in this case is:

$$\begin{cases} 10 + X = 14 \\ 16 + Y = 22 \rightarrow (2,2,2) \\ 20 + Z = 22 \end{cases}$$

Since the amount of resources for accomplishing activity (3-6) is not enough, it should be shifted to the decision point six. Decision point six at time (330,302,258,220) takes place at the end of activity (4-7).

$$R_6 = \{(3-6)\} \quad AR = (14,18,22)$$

As there is only one feasible activity at this stage, this activity is scheduled at this time.

$$S_{3,6} = (220,258,302,330) \quad F_{3,6} = (262,303,357,390)$$

By the end of this activity, only remaining activity (6-7) is scheduled. By the end of this activity, the project scheduling is completed.

$$S_{6,7} = (262,303,357,390) \quad F_{6,7} = (227,321,379,416)$$

Table 5. Scheduling Results Using the Proposed Algorithm

Activity	start time	End Time
(1-2)	(40.55.65.70)	(65.83.97.105)
(1-3)	(0.0.0.0)	(40.55.65.70)
(2-4)	(60.80.100.110)	(92.117.143.158)
(3-4)	(40.55.65.70)	(60.80.100.110)
(2-5)	(60.80.100.110)	(95.118.142.155)
(3-6)	(220.258.302.330)	(262.303.357.390)
(4-7)	(160.193.227.245)	(220.258.302.330)
(5-7)	(95.118.142.155)	(160.193.227.245)
(6-7)	(262.303.357.390)	(227.321.379.416)

Solving this example with shortage of resources and comparing its results with the results obtained by scheduling without resources indicates the importance of taking the resources into account in project scheduling.



CONCLUSION

In this paper, the activity durations and resource limitations are considered as fuzzy parameters. The heuristic parallel scheduling method was developed with fuzzy parameters, inspired by the improved fuzzy backward pass calculation method presented by Soltani and Haji [8], and using Wang ranking method [24]. To demonstrate the effectiveness of the proposed algorithm, an example with certain resource conditions was solved by the algorithm and the obtained solution had full consistency with the solution obtained by Bhaskar [22]. Finally, a numerical example was solved with fuzzy activity durations and resources. It was shown that the proposed algorithm is usable in problems with single resource type. For future studies it is recommended to investigate this algorithm for project scheduling problems with multiple types of resources. Other activity prioritizing rules can also be applied to this algorithm and the results be compared.

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